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Differential Angle Tracking for Close Geostationary Satellites

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For the determination of relative motion of two satellites located in close proximity in the geostationary orbit, a method of differential angle observation from a ground station is proposed. Observability of the relative orbital elements is proved, and the determination accuracy is analyzed on the basis of a linearized two-body orbital model. Experimental pseudodifferential angle observation indicates an accuracy of a few hundred meters for the determination of relative satellite position.

Nomenclature

A	= amplitude of diurnal angle variation
Az, El	= satellite azimuth and elevation; azimuth is measured clockwise from north
$a_{R,L,K}$	= relative perturbation accelerations
$c_1, \ldots, c_6$	= arbitrary constants
$E_1, \ldots, E_6$	= relative orbital elements
M	= mean periodical relative position error
N	= number of tracking measurements over one day
P	= Jacobian matrix, $\partial(u_1, \ldots, u_4)/\partial(E_1, \ldots, E_4)$
R, L, K	= relative position coordinates (see Fig. 1)
s	= time, measured by Earth rotation angle
T	= angle observation period
t	= time
u, v	= imaginary differential angle observations
x, z, Φ, H	= respectively, state vector, observation vector, state transition matrix, and observation matrix in Kalman filtering
α, ϵ	= differential azimuth and elevation
α_m	= modified differential azimuth, $\alpha \cos El$
β, γ	= angles of tracking geometry (see Fig. 1)
ρ	= satellite nominal range
ω	= Earth rotation rate
$(\ )'$	= $d(\ )/ds$

Introduction

BECAUSE of the increase in the use of geostationary orbits, it has become more and more common to have two or more satellites located close to each other in the orbit. Direct satellite broadcasting is a familiar case, where the satellites in primary-backup pairs from a number of nations or organizations are located at the same nominal longitudinal position of the orbit.[1] The situation also arises when replacing an old satellite with a new one while providing uninterrupted service, as well as for two partially failed communication satellites being paired to function as a full single satellite.[2] Future trends in space communications development will call for the orbital technique of keeping a number of clustered satellites in a specified formation to synthesize a large telecommunications system.[3] To operate these satellites without risk of collision or any other undesired mutual interference, or with adequate control for specified formation keeping, we must know their orbital motions, in particular the relative motions between satellites, as accurately as possible.[4]

One possible solution is to use upgraded tracking techniques such as multilateral ranging and optical tracking to provide improved accuracy for each satellite's orbit determination, but these have the obvious problems of extra workload and cost. Because our subject is the relative motion between satellites, there should be a method for relative satellite tracking that detects the relative motion in a direct manner.

This paper proposes a method for the relative orbit determination of closely located geostationary satellites by means of ground-based differential angle tracking. An angle-pointing antenna tracks the satellites in turn and the observed angles are differenced between the satellites. Because the error common to each observation is removed, an accurate relative orbit determination should be obtainable from the differential angles.

The standard methods of geostationary satellite tracking are ranging and angle observation, and theoretically they are equally usable for differential tracking. Under actual conditions, however, we find some difficulty in differential ranging. A ranging system is not supposed to have signal compatibility with all the satellites, particularly if they are from different organizations or nations. Should the ranging system be workable, the effect of differencing would not be perfect because ranging signals travel through communication equipment of separate satellites. Differential angle observation has, on the contrary, no such satellite-dependent problem; it requires only that an antenna should be able to track each satellite's beacon signal. Thus differential angle observation will be feasible in many cases where differential ranging is infeasible. Continuous angle observation is possible from passive signal reception, allowing operational simplicity and economy. Considering these realistic advantages, we select in this paper the method of differential angle observation.

First the differential tracking procedure is defined and the observability of the relative orbital elements is proved. Next the theoretical accuracy of the relative orbit determination is analyzed on the basis of first-order, two-body orbital dynamics. The error statistics of the differential angle observation are studied, which allows an estimate to be made of the actual accuracy of the relative orbit determination. Finally a filtering technique for the determination is briefly discussed.

Differential Angle Observation

Although angle observation is advantageous because of its wider applicability and continuous availability, it is subject to various environmental errors. Thermal deformations in the antenna structure under sunlight, fluctuations in the angle of arrival of the satellite radio wave due to turbulence in the atmosphere, and wind pressure all give rise to angle tracking observation error.[5,6] As a result the absolute accuracy of the angle observation cannot be better than a few thousandths of a degree. Our interest is in removing these errors by differential observations.

Suppose we have two satellites closely placed in the geostationary orbit, which we call the main satellite and subsatellite,

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and have a tracking antenna at a ground station with which the azimuth and elevation angles of the satellites are measured. The antenna is operated first to track the main satellite, next switched to the subsatellite and once again to the main satellite, thus making three successive angle observations for an identical time period of T, which is typically a few minutes. Taking the average for each observation gives three sets of angle data $(Az, El)_1$, $(Az, El)_2$, and $(Az, El)_3$. If the time needed for switching from the main to the subsatellite and back are the same, and the switching time is negligibly small, the differential angle of the subsatellite with respect to the main is given by

$$\begin{pmatrix}\alpha\\ \epsilon\end{pmatrix} = \begin{pmatrix}Az\\ El\end{pmatrix}_2 - \frac{1}{2}\left[\begin{pmatrix}Az\\ El\end{pmatrix}_1 + \begin{pmatrix}Az\\ El\end{pmatrix}_3\right] \tag{1}$$

as at the midpoint of the observations. Because the satellites are tracked with the same antenna, an error that is constant or varies linearly with time for the time of $3T$ is removed. Errors still present after differential processing will be due to short-period atmospherical fluctuations and wind effects. We collect the differential angles in this way to make the relative satellite tracking data.

This method can be applied without problem if the satellites emit their beacons in the same frequency band. If they do not use the same frequency band, a multiband antenna is needed with a careful calibration for the interband alignment of the electrical axes.

If the satellite azimuth and elevation angles were to vary linearly with time, Eq. (1) includes no error. Because this is not the case, and the angles vary like a diurnal sinusoid $A \sin \omega t$, the procedure has an error of

$$A \sin \omega t - (A/2)\left[\sin \omega(t-T) + \sin \omega(t+T)\right] \tag{2}$$

which takes the maximum of $A(\omega T)^2/2$. The observation switching time T must be sure that this error remains sufficiently small.

Observability Examination

We first show that the parameters defining the satellites' relative motion can be determined from the differential angle observations. We describe the relative motion of the subsatellite as observed from the main satellite in a coordinate system with its origin at the main satellite and the axes directed along radial R, longitudinal L, and orbit-normal K directions, as shown in Fig. 1. If we assume that the satellites are kept so close that they are confined within the same stationkeeping box of ± 0.1 deg, which compared with the orbital radius of 42,165 km is very small, the relative motion dynamics are approximated by the well-known equations of linearized relative orbital motion,[7] which for our coordinate system become

$$\begin{aligned} R'' - 2L' - 3R &= a_R \\ L'' + 2R' &= a_L \\ K'' + K &= a_K \end{aligned} \tag{3}$$

That is, time is measured by the Earth rotation angle s in radians instead of by the usual time t in seconds. The right-hand sides are the relative perturbation accelerations between the satellites. Assuming a two-body problem, we can write a general solution for Eqs. (3) as the following:

$$\begin{aligned} R &= -(2/3)E_2 + E_3 \cos s + E_4 \sin s \\ L &= E_1 + E_2 s - 2E_3 \sin s + 2E_4 \cos s \\ K &= E_5 \cos s + E_6 \sin s \end{aligned} \tag{4}$$

The arbitrary parameters $E_1, \ldots, E_6$ define the relative orbital motion of the subsatellite observed from the main, which we call the relative orbital elements. E_1 and E_2 are mean satellite longitudinal separation and its drift rate, E_3 and E_4 correspond to relative eccentricity, and E_5 and E_6 to relative inclination. In first-order modeling, the relative orbit determination is the problem of determining the six relative orbital elements.

We describe the geometry of the ground station and the satellites as in Fig. 1, where the Earth is assumed to be ideally spherical. A small deviation in the relative satellite position gives rise to a change in differential observables by

$$\begin{bmatrix}\delta\alpha_m\\ \delta\epsilon\end{bmatrix} = (1/\rho)\begin{bmatrix}0 & -\cos\gamma & \sin\gamma\\ \sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma\end{bmatrix}\begin{bmatrix}\delta R\\ \delta L\\ \delta K\end{bmatrix} \tag{5}$$

where we introduce $\alpha_m = \alpha \cos El$ in place of α to resolve the problem of singularity when El is almost a right angle. This is a formula taken from Ref. 8, which has been modified for the differential form.

The observability of the relative orbit determination is examined as follows: We form the partial derivatives of the observables α_m and ϵ with respect to the unknown orbital elements $E_1, \ldots, E_6$ using Eqs. (4) and (5), and put

$$\sum_{i=1}^{6} c_i \frac{\partial \alpha_m}{\partial E_i} = 0 \qquad \sum_{i=1}^{6} c_i \frac{\partial \epsilon}{\partial E_i} = 0$$

If and only if these equations lead to $c_1 = \cdots = c_6 = 0$, namely $\partial(\alpha_m, \epsilon)/\partial E_i$, $i = 1, \ldots, 6$ are mutually independent as functions of time, are the elements determined from the (α_m, ϵ) observations. If we make the tracking observation over one day (from $s = 0$ to 2π) then constant, s, $\cos s$, and $\sin s$ are regarded as independent functions of time, so that their coefficients all have to be zero.[8] Hence we obtain eight sets of equations for c_i, which are summarized as

$$\begin{bmatrix}-\cos\gamma & 0\\ 0 & -\cos\gamma\\ \cos\beta\sin\gamma & -2/3\cdot\sin\beta\\ 0 & \cos\beta\sin\gamma\end{bmatrix}\begin{bmatrix}c_1\\ c_2\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \tag{6a}$$

and

$$\begin{bmatrix}0 & -2\cos\gamma & \sin\gamma & 0\\ 2\cos\gamma & 0 & 0 & \sin\gamma\\ \sin\beta & 2\cos\beta\sin\gamma & \cos\beta\cos\gamma & 0\\ -2\cos\beta\sin\gamma & \sin\beta & 0 & \cos\beta\cos\gamma\end{bmatrix} \times \begin{bmatrix}c_3\\ c_4\\ c_5\\ c_6\end{bmatrix} \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \tag{6b}$$

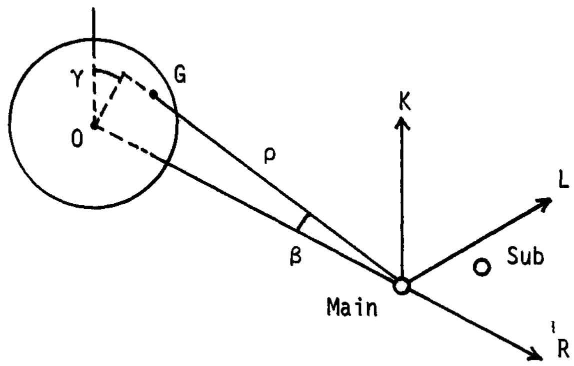

Fig. 1 Relative satellite position coordinates: R = radial, L = longitudinal, K = orbit normal components, and the tracking geometry. G is the ground station, O is the Earth center, ρ is the nominal satellite range at G, and γ is the angle between the Earth axis and the plane in which O, G, and the main satellite lie.

Because the tracking station is on the Earth's surface, β is confined to $|\beta| \leq 8.7$ deg, and therefore the two column vectors in the coefficient matrix of Eq. (6a) are independent, from which $c_1 = c_2 = 0$ results. In Eq. (6b), the determinant of the coefficient matrix is $4\cos^2\beta + \sin^2\beta \sin^2\gamma \geq 4\cos^2\beta \geq 3.91 \neq 0$, which implies $c_3 = \cdots = c_6 = 0$. Thus we have $c_1 = \cdots = c_6 = 0$, so that the differential orbital elements are assured of being fully observable from one-day tracking, irrespective of the tracking station location.

Theoretical Accuracy of the Determination

Using the first-order orbital model, we can derive the relation between the relative orbit determination accuracy and the differential observation quality. Considering that β is limited by $|\beta| \leq 8.7$ deg while γ is arbitrary, we approximate as $\beta = 0$, which makes subsequent analysis straightforward. Because of the assured observability, the change in the error analysis due to the approximation would not be drastic. Equation (5) becomes

$$\begin{bmatrix} -\delta\alpha_m \\ \delta\epsilon \end{bmatrix} = (1/\rho) \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} \delta L \\ \delta K \end{bmatrix} \tag{7}$$

We introduce imaginary observables u and v by

$$\begin{bmatrix} \delta u \\ \delta v \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} -\delta\alpha_m \\ \delta\epsilon \end{bmatrix} \tag{8}$$

from which Eq. (7) becomes as simple as

$$\begin{bmatrix} \delta u \\ \delta v \end{bmatrix} = (1/\rho) \begin{bmatrix} \delta L \\ \delta K \end{bmatrix} \tag{9}$$

where u and v are directly measuring the relative motion components L and K.

From Eqs. (4) and (9) we obtain the following partial derivatives:

$$\frac{\rho\partial u}{\partial E_1} = 1, \quad \frac{\rho\partial u}{\partial E_2} = s, \quad \frac{\rho\partial u}{\partial E_3} = -2\sin s$$
$$\frac{\rho\partial u}{\partial E_4} = 2\cos s, \quad \frac{\partial u}{\partial E_5} = 0, \quad \frac{\partial u}{\partial E_6} = 0 \tag{10a}$$

$$\frac{\partial v}{\partial E_1} = 0, \quad \frac{\partial v}{\partial E_2} = 0, \quad \frac{\partial v}{\partial E_3} = 0$$
$$\frac{\partial v}{\partial E_4} = 0, \quad \frac{\rho\partial v}{\partial E_5} = \cos s, \quad \frac{\rho\partial v}{\partial E_6} = \sin s \tag{10b}$$

Equations (10a) and (10b) show that we have two separate problems; one is to determine $E_1, \ldots, E_4$ from observed u, and the other E_5 and E_6 from v. We make the observations u_i and v_i over one day at the times of $s_i = 2\pi i/N$, $i = 0, 1, \ldots, N$. Consider first the determination of $E_1, \ldots, E_4$. We assume that the observation error is stationary, with constant σ_u and σ_v, during the tracking. According to least square theory, the covariance of the determination errors $\delta E_1, \ldots, \delta E_4$ is given by the matrix

$$E\left\{ \begin{bmatrix} \delta E_1 \\ \delta E_2 \\ \delta E_3 \\ \delta E_4 \end{bmatrix} (\delta E_1, \delta E_2, \delta E_3, \delta E_4) \right\} = (P^T P)^{-1}\sigma_u^2$$

where $E\{..\}$ is for the expected value and P is the Jacobian matrix with $P_{ij} = \partial u_i/\partial E_j$. From Eq. (10a) it follows immediately that

$$(P_{i1}, \ldots, P_{i4}) = (1/\rho)(1, s_i, -2\sin s_i, 2\cos s_i)$$

If N is large enough, matrix calculations lead to

$$P^T P \approx \frac{N}{\rho^2} \begin{bmatrix} 1 & \pi & 0 & 0 \\ \pi & 4\pi^2/3 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

where approximations are made; for instance, its 2, 3-element is being derived as

$$-2\sum_{i=0}^{N} \frac{2\pi i}{N} \sin\frac{2\pi i}{N} \approx -\frac{N}{2\pi}\int_0^{2\pi} x \sin x \, \mathrm{d}x = 2N$$

Applying the same procedure for the determination of E_5 and E_6, we obtain the following error covariance matrices:

$$E\left\{ \begin{bmatrix} \delta E_1 \\ \delta E_2 \\ \delta E_3 \\ \delta E_4 \end{bmatrix} (\delta E_1, \delta E_2, \delta E_3, \delta E_4) \right\}$$
$$= \frac{\rho^2\sigma_u^2}{N} \begin{bmatrix} 8.58 & -2.41 & 2.41 & 0 \\ -2.41 & 0.769 & -0.769 & 0 \\ 2.41 & -0.769 & 1.27 & 0 \\ 0 & 0 & 0 & 0.50 \end{bmatrix} \tag{11a}$$

$$E\left\{ \begin{pmatrix} \delta E_5 \\ \delta E_6 \end{pmatrix} (\delta E_5, \delta E_6) \right\} = \frac{2\rho^2\sigma_v^2}{N} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{11b}$$

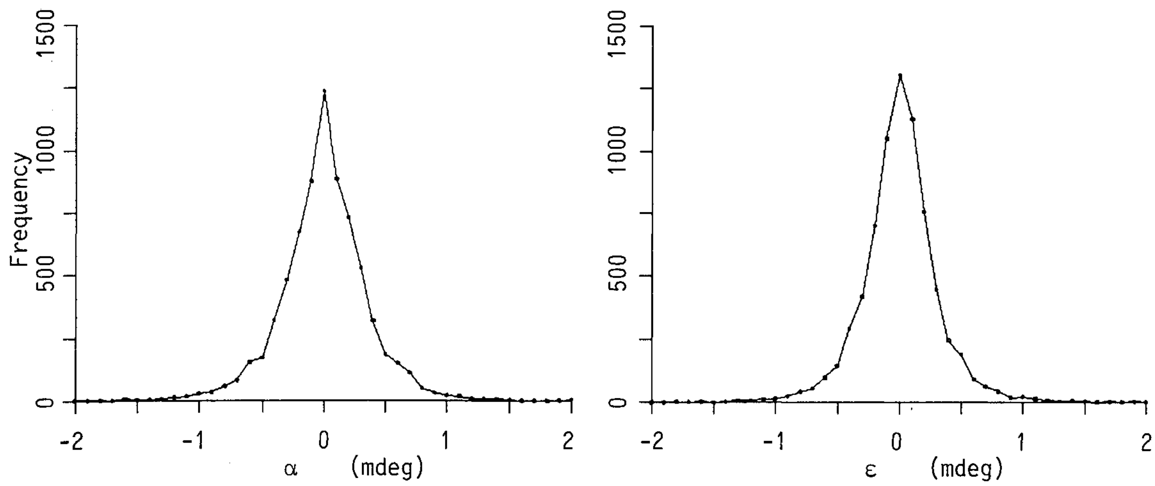

Fig. 2 Distribution of pseudodifferential angles for 1 yr ($\sigma_\alpha = 0.39$ mdeg, $\sigma_\epsilon = 0.27$ mdeg).

From Eq. (11a) we have σ_{E_1} and σ_{E_2} to indicate the determination errors in mean satellite separation and relative drift rate, which are easily understood. The effects of the errors in $E_3, \ldots, E_6$ are summed up as follows. These errors give rise to periodical errors in the determined relative position by

$$\delta R = \delta E_3 \cos s + \delta E_4 \sin s$$

$$\delta L = -2\delta E_3 \sin s + 2\delta E_4 \cos s$$

$$\delta K = \delta E_5 \cos s + \delta E_5 \sin s$$

from which mean periodical error M is evaluated as

$$M^2 = E\left\{\frac{1}{2\pi}\int_0^{2\pi} (\delta^2 R + \delta^2 L + \delta^2 K)\ \mathrm{d}s\right\}$$
$$= \tfrac{1}{2}\left(5\sigma_{E_3}^2 + 5\sigma_{E_4}^2 + \sigma_{E_5}^2 + \sigma_{E_6}^2\right)$$

Thus we have three error indices for the relative orbit determination performance:

Mean satellite separation error $\quad \sigma_{E_1} = 2.93\rho\sigma_u/\sqrt{N}$

Mean relative drift error (per day) $\quad 2\pi\sigma_{E_2} = 5.51\rho\sigma_u/\sqrt{N}$

Mean periodical error $\quad M = \rho\sqrt{4.43\sigma_u^2 + 2\sigma_v^2}/\sqrt{N}$

(12)

where the observation errors for u and v are converted, following Eq. (8), as

$$\sigma_u^2 = \sigma_{\alpha_m}^2 \cos^2\gamma + \sigma_\epsilon^2 \sin^2\gamma$$
$$\sigma_v^2 = \sigma_{\alpha_m}^2 \sin^2\gamma + \sigma_\epsilon^2 \cos^2\gamma \tag{13}$$

with $\sigma_{\alpha_m} = \sigma_\alpha \cos El$, provided that the observation errors in α and in ϵ are uncorrelated.

Observation Error Statistics

To evaluate the relative orbit determination accuracy with Eqs. (11) or (12), we must know the error statistics of the differential tracking observation.

We can make the error statistics if we have angle observation data from one satellite. Suppose we make an angle observation of a geostationary satellite over a time period of $3T$. We divide the observation equally into three observations of period T. Taking the average in each gives three data sets: $(Az, El)_1$, $(Az, El)_2$, and $(Az, El)_3$, from which we obtain the differential angle by Eq. (1). The differential angle (α, ϵ) obtained, which we should call a pseudodifferential angle, would always be zero, except for the error of Eq. (2), if there were no observation noise. Because the angle observation does include noise, the pseudodifferential angle provides a sample of the error that will arise in the actual differential angle observation for two separate satellites. If the pseudodifferential angles are collected in this way for a long term with various environmental conditions being included, it will provide the observation error model for the actual case of the relative tracking of two satellites.

Communications Research Laboratory (CRL) has previously conducted a study on the atmospheric propagation of satellite beacon signals in relation to an experimental communication satellite,[9] where a data base of satellite angle observations has been organized. The satellite beacon at 19.5 GHz is tracked in monopulse by a 13-m-diam antenna at the Kashima station of CRL, with $El = 48$ deg. The relevant parameters of tracking geometry are $\rho = 37{,}220$ km and $\gamma = 8.2$ deg in this case. The antenna was designed as a conventional satellite communication antenna without any special consideration for angle measurement accuracy. Pseudodifferential angles were sampled out for 1 yr from the data base at intervals of 1 h with $T = 3$ min. The distribution of the pseudodifferential angles (α, ϵ) appears to be near Gaussian as shown in Fig. 2. The error due to Eq. (2) in this case is less than 0.01 mdeg. Considering that the relative orbit determination is based on one-day tracking, σ_α and σ_ϵ were evaluated for each day as in Fig. 3. It is worth noticing that the peaks of σ_α and σ_ϵ sometimes appear on the same day and sometimes not. This fact suggests that a part of the differential angle error is due to wind pressure, whose effect depends on the wind direction with respect to the antenna azimuth. From Fig. 3, cumulative distribution of the daily error σ values was plotted in Fig. 4. The error model thus obtained is attributed to a particular set of antenna, its site location, and the receiving frequency, but this is the only error model that we know of at the moment. Using this error model, the relative orbit determination accuracy was estimated by Eqs. (12) and (13) with $N = 24$, as in Table 1. We can expect an accuracy of a few hundred meters for the determination of relative satellite position.

In addition to the long-term (α, ϵ) data, the data were collected for a few days with continuous sampling every 9 min to check auto/cross correlations of α and ϵ, and it was shown that they are uncorrelated white noises. The assumption of Eq. (13) was therefore valid, and we can expect further error reduction if we choose a larger N, up to 160 for the present case of $T = 3$ min, in Eqs. (12).

Filtering Technique

Because the differential angle is continuously available without much extra work at the tracking station, real-time processing by a Kalman filter is suitable for the relative orbit determination to continuously monitor the relative satellite motion. An orthodox design for the processing filter would be to use two separate orbital models, each of which would consider all relevant perturbations, but this would require a large computer program. A simplified filter is possible, on the contrary, if we use the relative orbital model for the two satellites.

The state vector and the observation vector are defined by

$$x = (R', L, K, R', L', K')^T \text{ and } z = (\alpha, \epsilon)^T$$

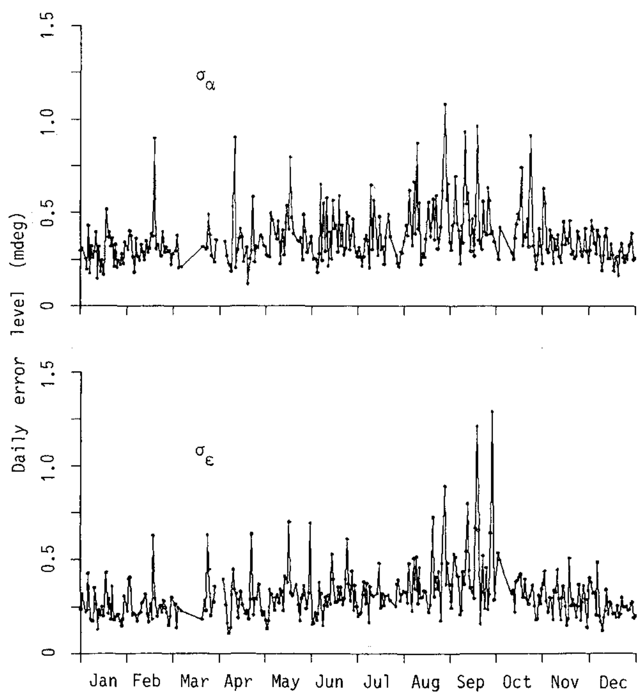

Fig. 3 Day-to-day σ evaluations of pseudodifferential angles.

The state transition matrix is made from Eqs. (3) as

$$\Phi(s) = \begin{bmatrix} 4-3\cos s & 0 & 0 & \sin s & 2-2\cos s & 0 \\ 6\sin s - 6s & 1 & 0 & 2\cos s - 2 & 4\sin s - 3s & 0 \\ 0 & 0 & \cos s & 0 & 0 & \sin s \\ 3\sin s & 0 & 0 & \cos s & 2\sin s & 0 \\ 6\cos s - 6 & 0 & 0 & -2\sin s & 4\cos s - 3 & 0 \\ 0 & 0 & -\sin s & 0 & 0 & \cos s \end{bmatrix}$$

Table 1 Relative orbit determination performance by differential angle tracking

	Confidence level		
Error index	99%	90%	80%
Satellite separation, km	0.24	0.15	0.11
Relative satellite drift, km/day	0.46	0.27	0.21
Mean periodical error, km	0.23	0.13	0.11

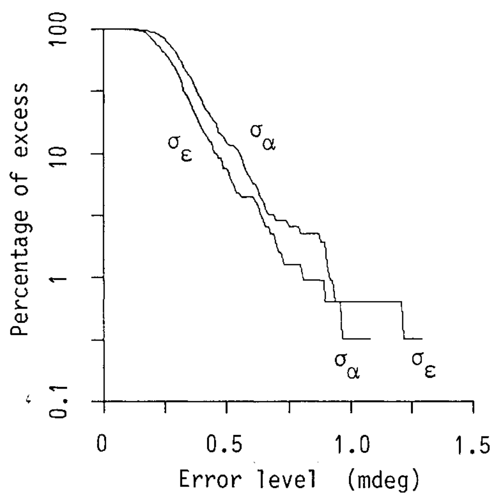

Fig. 4 Cumulative distribution of the daily σ. The probability of exceeding a given error level is plotted.

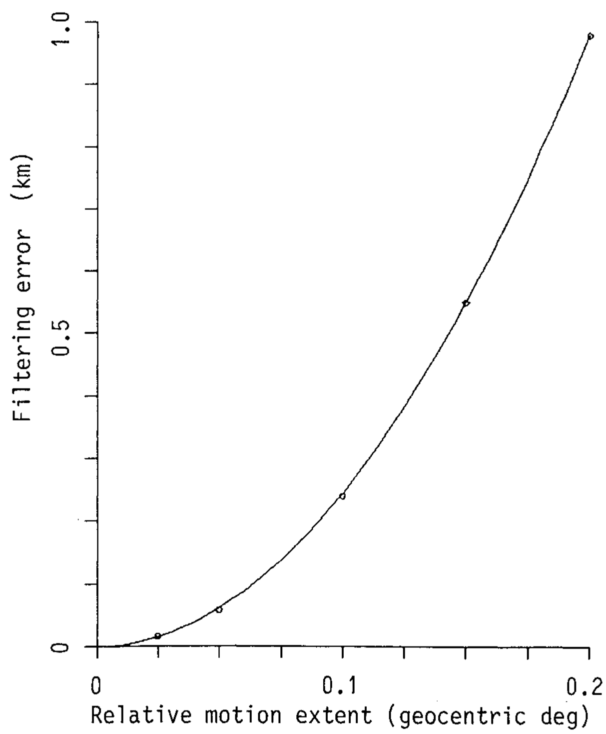

Fig. 5 Filter modeling error. Small circles are from the simulation test, to which a quadratic curve is fitted.

and the observation matrix is

$$H = \begin{bmatrix} \dfrac{\partial Az}{\partial R} & \dfrac{\partial Az}{\partial L} & \dfrac{\partial Az}{\partial K} & 0 & 0 & 0 \\ \dfrac{\partial El}{\partial R} & \dfrac{\partial El}{\partial L} & \dfrac{\partial El}{\partial K} & 0 & 0 & 0 \end{bmatrix}$$

where Az and El are for the subsatellite, and the partial derivatives are evaluated at its nominal position. As it is difficult in many cases to know the correct area/mass ratios of the satellites, and there may be any combination of satellites of different design, the relative solar radiation pressure must be considered as one of the state vector components. As to other perturbations relevant to geostationary orbits, which are lunar-solar gravity and Earth gravity harmonics, their effect upon the relative motion was evaluated to be small by a numerical test. Assuming a two-satellite geometry with a longitudinal separation of 0.1 deg and a relative inclination of 0.1 deg, 1-day evolution of the satellite geometry was numerically generated with and without the perturbations. The relative geometry distortion due to the perturbations was less than 40 m in each orthogonal component, which is smaller than the observation noise effects in Table 1, so that these perturbations may be disregarded. That is to say, first-order modeling is valid. A linearized Kalman filter is composed in this concept[10,11] resulting in a program of a few hundred Fortran statements that can run on a microcomputer.

The performance of the filter was tested, where a numerical orbit generator makes the orbits of two supposed satellites and their Az, El are differenced so as to simulate the differential tracking data. First the filter response on the simulated error-free observation input was tested. The satellite geometry was set as $(R,L,K)=(0,\ d,\ d\sin s)$, namely the satellites have a relative north-south libration motion at a constant east-west separation, with d measuring the extent of the relative motion. The filtering error, which is the relative position error still existing after the filter has converged to a steady state, is plotted against d in Fig. 5. Because of the linearized relative model, the filtering error is dependent on the extent of relative motion. Thus we know that the determination error is a superposition of the error indicated by Eqs. (12) and that by Fig. 5 if we use the linearized relative filter.

The performance test was made again, this time using the simulated tracking data with observation noises added, while d was set to zero, to see the relation between the relative position determination error and the observation noise level. The results were in agreement with Eqs. (12); therefore, the error sensitivity analysis in the section entitled Theoretical Accuracy of the Determination was validated.

Concluding Summary

The method of differential angle observation was proposed for the relative orbit determination of two satellites closely located in the geostationary orbit. The determination accuracy was theoretically formulated in terms of observation error level. An observation error model was made from experimental pseudodifferential angle data, which led to an estimation of the accuracy of a few hundred meters for the relative satellite position. It should be noted that the accuracy estimation derived as such has been based on some simplifications; in particular the pseudodifferential observation error statistics may have overlooked some minor error factors that would appear in real observations, and only a specially organized tracking experiment could answer the question. But this is not to vitiate the basic conclusion that the concept of differential angle tracking is effective for accurate relative orbit determinations. The method for conducting relative tracking is obvious if we have three or more satellites, while it may be necessary to prepare a well-calibrated antenna to cope with multiband satellite beacons.

If the satellites are close enough that they stay within the same stationkeeping box of ± 0.1 deg, the linearized simple Kalman filter is a suitable choice for the determination processing. Otherwise, if an accuracy of a few hundred meters is pursued irrespective of the extent of the satellite motion, the filter must be fully modeled with separate orbit dynamics for the two satellites.

The proposed method of relative orbit determination will be usable for collision avoidance control in a crowded geostationary orbit, as well as for formation control of the geostationary cluster satellites.

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